steel ingots each weighing 7 mg, at various times after casting. The hardened ingots were then rolled into a square billet of cross section 120 × 120 mm^2. Autoradiograms (using the method described previously [1-3]) of the transverse templets selected at 5% intervals along the length of the billet were taken; from these autoradiograms the depth of penetration of the isotope into the ingot was determined. The depth of penetration of the isotope as a function of the time the additive was introduced is linear throughout the range studied [12].

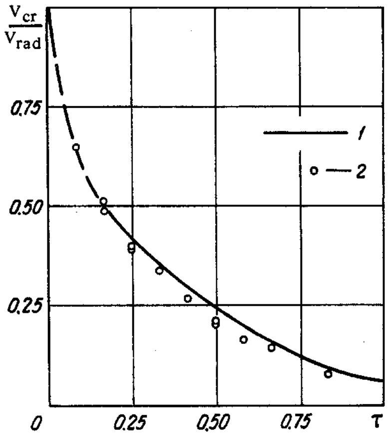

Fig. 2. Change in time of convective metal volume (τ is the time (beginning from the time casting begins) in hours): 1) theoretical curve; 2) experimental data obtained by the radioactive indicator method.

By knowing the change in depth of penetration of the isotope with time we can calculate the change in volume of the convective metal V_{cr}. The theoretical curve (9) is compared with the experimental data obtained for the change in time of the volume of convective metal V_{cr} (Fig. 2). There is almost complete agreement. The solution corresponding to (9) at the boundary in Fig. 2 begins 10 minutes after casting is ended; the dashed line represents the change in the ratio of the liquid-metal volume V_{cr} to the volume of the mold V_{rad} during the first stage of the process when the ingot is still overheated.

NOTATION

$\bar{\alpha}$ is the heat transfer coefficient averaged over the height of the ingot; θ is the temperature head in a horizontal ingot cross section and is equal to the difference in temperature at the center of the ingot and at the boundary of the hardened crust; S_{cr} is the surface area common to the circulating metal and hardened crust; V_{cr} is the volume of circulating metal; c is the specific heat of the melt; γ is the specific gravity of the melt; ρ is the heat of crystallization; τ is the time; h is the height of the circulating-metal zone; β is the coefficient of thermal expansion; g is the gravitational acceleration; ν is the kinematic viscosity; and λ is the coefficient of heat conductivity.

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POLYTROPIC EXPONENT OF COMPRESSION OF A MOIST GAS

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Papers [1-4] dealt with the effect of moistening of air on the characteristics of compression in the absence of external heat transfer and on the following assumptions: The evaporated moisture and the air have the same specific heat; the heating of the liquid during compression is negligible; and the exponent n is constant during compression. The vapor formed by evaporation of the moisture is assumed to be saturated.

We consider a thermally homogeneous thermodynamic system composed of an ideal gas and a liquid uniformly dispersed throughout the volume of the system. The liquid drops are internal heat sinks. The system interacts thermally and mechanically with the surrounding medium. We write the heat balance for the elementary polytropic compression of the system with the instantaneous value of the exponent n:

$$\delta Q_n = \delta Q_r + \delta Q' + \delta Q_0, \tag{1}$$

where $\delta Q_n = (M_1 c_{V_1} + M_2'' c_{V_2})(k - n/n - 1)dT$ is the heat of the polytropic process.

Assuming that the dispersed moisture does not significantly affect heat exchange with the surrounding medium we write the expression for δQ_0 in terms of the polytropic exponent of compression n_0 of the dry gas

$$\delta Q_0 = (M_1 c_{V_1} + M_2'' c_{V_2}) \frac{k_0 - n_0}{n_0 - 1} dT,$$

where k_0 is the adiabatic exponent of the dry gas.

Substituting the expressions for δQ_n, δQ_r, $\delta Q'$ and δQ_0 in (1) and introducing the symbols $x = M_2''/M_2$, the relative concentration of evaporated moisture, $\mu = M_2/M_1$, the initial moisture content of the gas, we obtain

$$(c_{V_1}+\mu x c_{V_2})\frac{k-n}{n-1}=\mu r\frac{dx}{dT}+$$

$$+\mu c(1-x)+(c_{V_1}+\mu x c_{V_2})\frac{k_0-n_0}{n_0-1}. \qquad (2)$$

Henceforth we will confine ourselves to a consideration of the case $\mu \ll 1$. In this case $k \approx k_0$ [5]. We rewrite Eq. (2) in the form

$$(c_{V_1}+\mu x c_{V_2})\frac{n_0-n}{n-1}=\mu r\frac{dx}{dT}+\mu c(1-x), \qquad (3)$$

neglecting $\delta n = k - n_0$ in comparison with n_0. To determine dx/dT we will assume that the vapor saturates the volume of the system and compression is confined to the instant of complete evaporation of the moisture. With these assumptions we can write the Clapeyron-Clausius system of equations for the vapor and the equations of state of an ideal gas:

$$\frac{dp_2}{dT}=\frac{r}{T(v''-v')}, \qquad (4)$$

$$p_2V=M_2''R_2T. \qquad (5)$$

Neglecting in expression (4) the value of the specific volume of the water in comparison with the specific volume of the dry saturated vapor $v'' = V/M_2''$ and using the previously adopted symbols, we obtain

$$\frac{dp_2}{dT}=\frac{M_2rx}{TV}, \qquad (6)$$

$$p_2V=M_2R_2Tx. \qquad (7)$$

The system of equations (6) and (7) gives the temperature dependence of the saturation pressure p_2 in the form

$$p_2=p_0\exp\left(-\frac{r}{R_2T}\right). \qquad (8)$$

From a joint consideration of (6)–(8) with due regard to the relationship between the thermodynamic parameters of the polytropic process $dV/dT = -V/T(n-1)$ we obtain an expression for dx/dT:

$$\frac{dx}{dT}=\frac{x}{T}\left(\frac{r}{R_2T}-\frac{1}{n-1}-1\right). \qquad (9)$$

From Eqs. (3) and (9) we find the instantaneous value of the polytropic exponent n:

$$n=\left[n_0+\frac{r^2\mu x}{R_2T^2(c_{V_1}+\mu x c_{V_2})}+\right.$$

$$\left.+\frac{\mu c(1-x)}{c_{V_1}+\mu x c_{V_2}}\right]\left[1+\frac{r^2\mu x}{R_2T^2(c_{V_1}+\mu x c_{V_2})}+\right.$$

$$\left.+\frac{\mu c(1-x)}{c_{V_1}+\mu x c_{V_2}}-\frac{r\mu x}{(c_{V_1}+\mu x c_{V_2})T}\right]^{-1}. \qquad (10)$$

We determine the mean integral value of the polytropic exponent $\bar{n}$ in the ranges $0 \le x \le 1$ and $T_1 \le T \le T_2$:

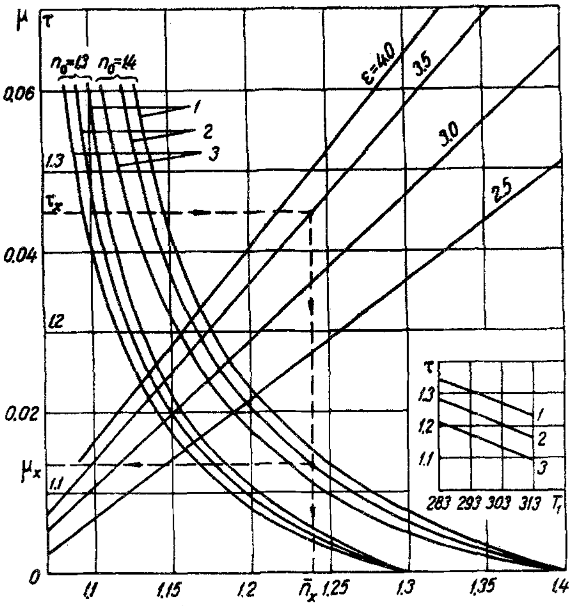
$$\bar{n}=1+\frac{n_0-1}{\dfrac{r^2\mu}{R_2T^2c_V^*}-\dfrac{r\mu}{T_2c_V^*}-\dfrac{\mu c}{c_{V_1}}}\times$$

$$\times\ln\left[\left(1+\frac{r^2\mu}{R_2T_2c_V^*}-\frac{r\mu}{T_2c_V^*}\right)\Big/\left(1+\frac{\mu c}{c_{V_1}}\right)\right], \qquad (11)$$

where $c_V^* = c_{V_1} + \mu c_{V_2}$.

It follows from (11) that $\lim\limits_{\mu\to 0}\bar{n}=n_0$, which corresponds to compression of a dry gas; $\lim\limits_{\mu\to\infty}\bar{n}=1$ (isothermal compression). Thus, $1 < n \le \le n_0$.

The figure shows the results of using Eq. (11) to calculate the compression of air with the injection of water. The nomogram gives the amount of injected moisture μ per kg of compressed air required for the attainment of a prescribed temperature T_2 °K at the end of compression for a given degree of compression ε and initial temperature T_1° K; the nomogram also gives the mean polytropic exponent $\bar{n}$ of the process. The calculations were carried out for the case of an adiabatically isolated system ($n_0 = k = 1.4$) and with due allowance for external heat transfer ($n_0 = 1.3$).



Nomogram for determination of main compression parameters of moist air: 1–3) T_2 = 383, 363, and 343° K, respectively.

The dashed line illustrates the use of the nomogram for the following conditions: temperature at start of compression T_1 = 298° K, required temperature at end of compression T_2 = 383° K, ratio of absolute temperatures at end and start of compression

$$\tau_x = T_2/T_1 = 1.28,$$

degree of compression ε = 3.5, polytropic exponent of dry (without injection) compression $n_0 = k = 1.4$ (no external heat transfer). The required amount of injected moisture in this case is μ_x = 0.013 kg per kg of compressed air. The mean polytropic exponent of compression is $\bar{n}_x$ = 1.244.

NOTATION

Q_n is the heat of polytropic process; M_1 is the mass of compressed gas; M_2'' is the mass of evaporated moisture; k is the adiabatic exponent; c_{V_1} is the specific heat of the gas; c_{V_2} is the specific heat of the vapor; $\delta Q_r = r dM_2''$ is the heat of evaporation of moisture; r is the heat of vaporization; $\delta Q' = (M_2 - M_2'')c dT$ is the heating of moisture during compression; c is the specific heat of fluid; M_2 is the initial mass of dispersed moisture; δQ_0 is the heat transferred between system and surrounding medium.

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